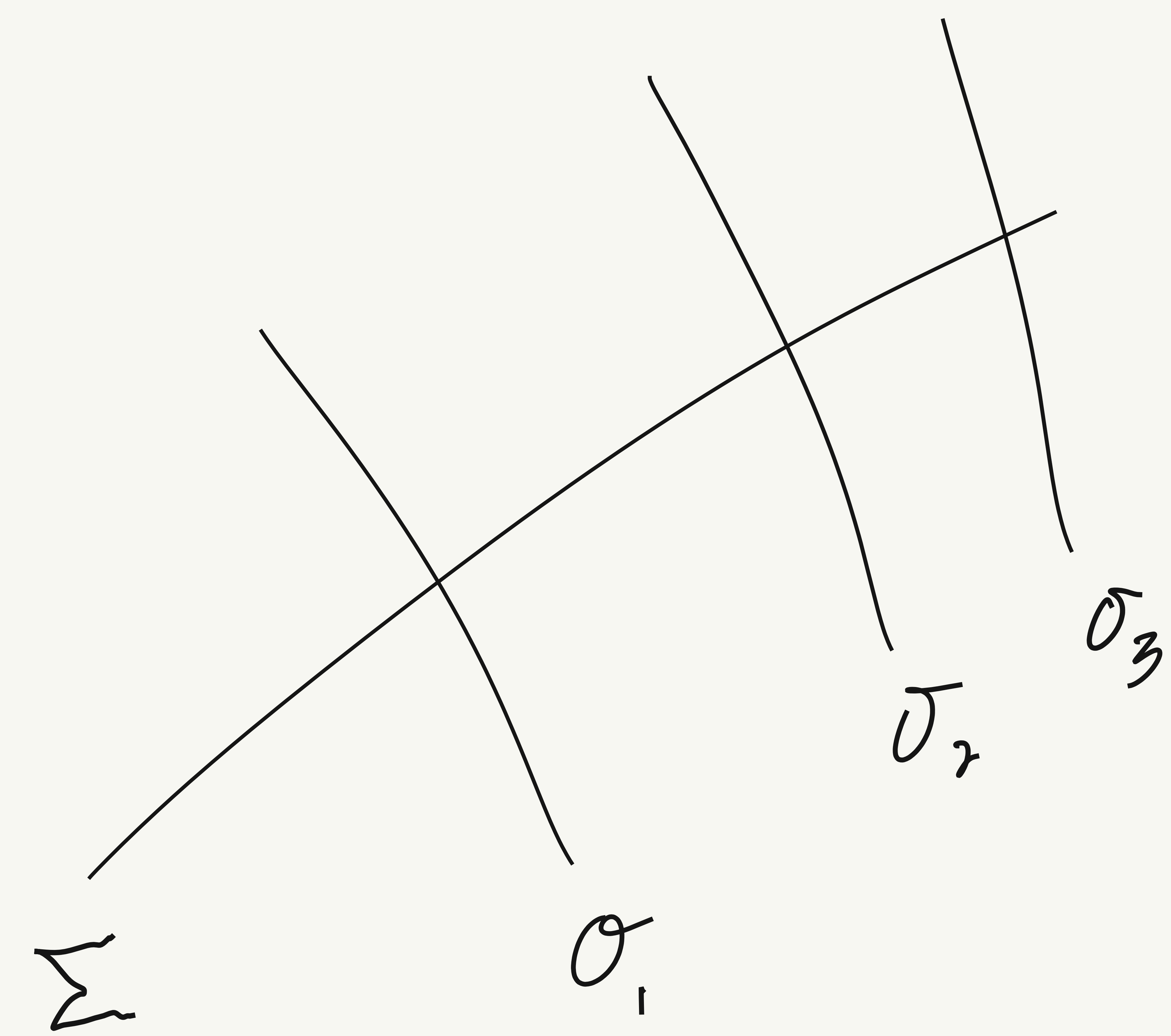
Solving Diffusion with Geometry and Max Cal

$$S[\gamma] = \int P(\gamma) \ln P(\gamma)$$



$$\underbrace{\dot{x} = f(x)}_{}$$

phase plane

(1) $\dot{x} = f(x)$

$\gamma = \{ x(t) \dots x(t_f) \}$

$\forall x, \; \gamma(x)$

$f(x)$

:=

tangent to

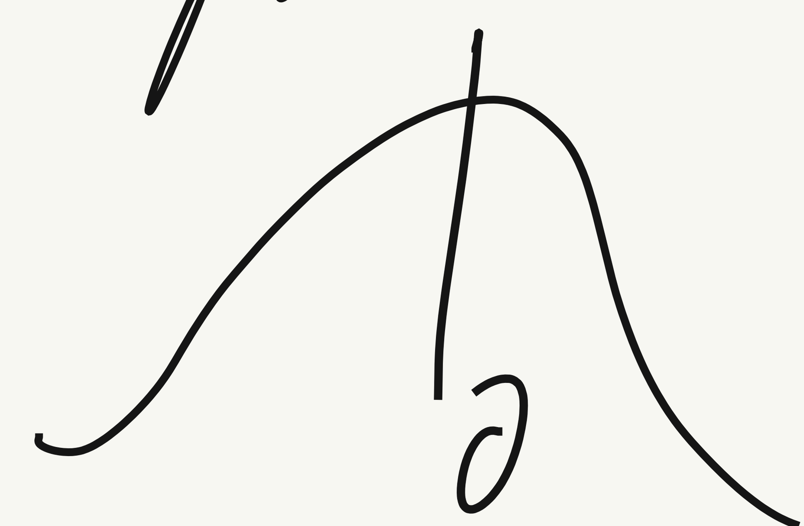
(2) $\dot{x} = f(x) + \eta(t)$ $\rightarrow$ $\int^t \eta \, ds$

W_t s.t.

γ

$P(\gamma)$

$$\underbrace{W_s - W_t}_{\eta(t)}$$



P ~~from~~ FP

$FP \Rightarrow Pr(\gamma)$

FP =

$$\frac{\partial \rho}{\partial t} = a \frac{\partial}{\partial x} \rho(x,t) + \ldots$$

$$L^{+}\rho = \dot{\rho}$$

Soln FP

$P(y)$

$$\min \left(\underbrace{\int L \, dt}_{S} \right)$$

$$\delta S = 0$$

$$\delta S = 0 \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

$$m \ddot{x} = F$$

$$\gamma(t) = \{ x(t) \ldots x(t_f) \}$$

$$x$$

y

$$\max \left(-\int P(x) \ln(P(x))\, dx \right)$$

$$\Rightarrow \max \left(-\int P(y) \ln(P(y))\, dy \right)$$

f

$$\Rightarrow \max \left(-\int P(y \mid f) \ln(P(y \mid f))\, dy \right)$$

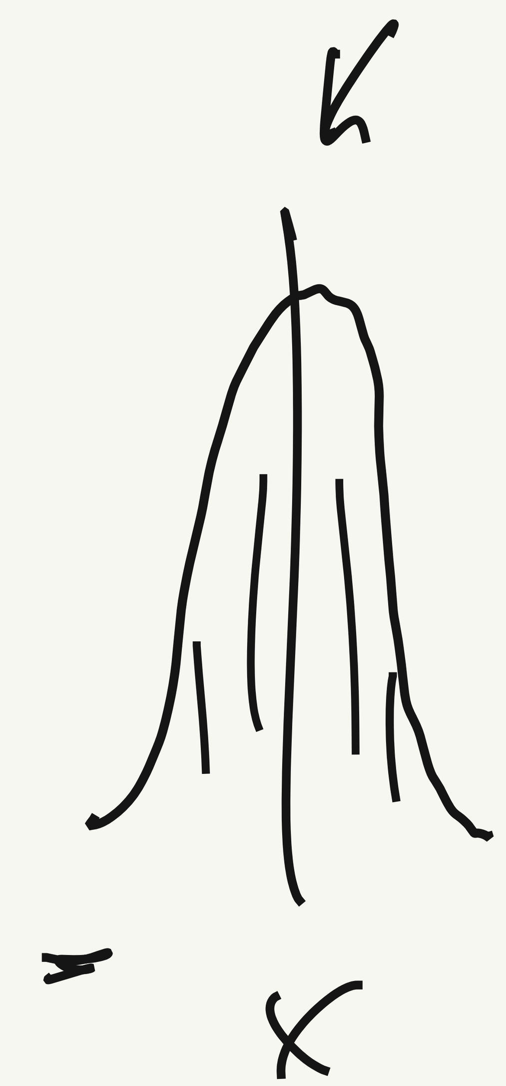
for f = constraints

$$-\int P(\eta|\gamma)\ln\left(\frac{P(\eta|\gamma)}{P(\eta)}\right) = S[\gamma]$$

$\Rightarrow$ relative entropy

$$\operatorname{argmax}(S[\eta]) = \underline{\underline{P(\eta)}}$$

$S[\gamma]$



$$S[\gamma] = \int P(\gamma|J) \ln(P(\gamma|J))\, d\gamma$$

$$S'[\gamma] = S[\gamma] + S[\lambda \cdot J] + \lambda_1 J_1 + \lambda_2 J_2 \ \dots$$

$$\lambda \cdot \int J(\gamma)\, P(\gamma)\, d\gamma - J$$

$$\frac{\partial S[\gamma]}{\partial P(\gamma)} = 0$$

$$\Rightarrow$$

$$\ln(\gamma) + \lambda \cdot J = 0$$

$$\Rightarrow$$

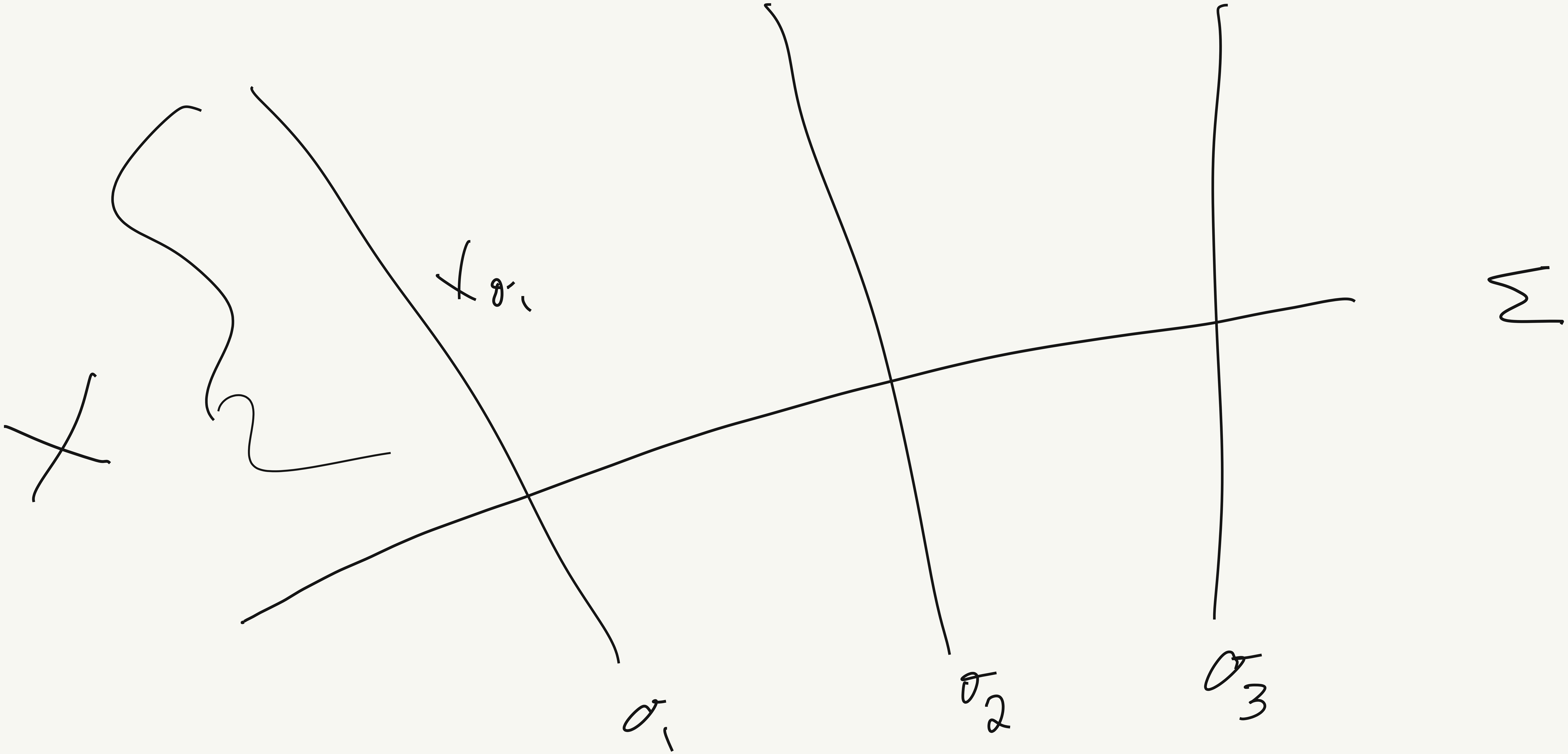
$$\gamma = \exp\{-\lambda \cdot J\}$$

$$\lambda \int J(y)\, P(y)\, dy$$

$$J = \frac{E}{T} \qquad \lambda^{-1} = k_B$$

$$\ln(y) + \lambda J \Longrightarrow y = \exp\{-\lambda J\}$$

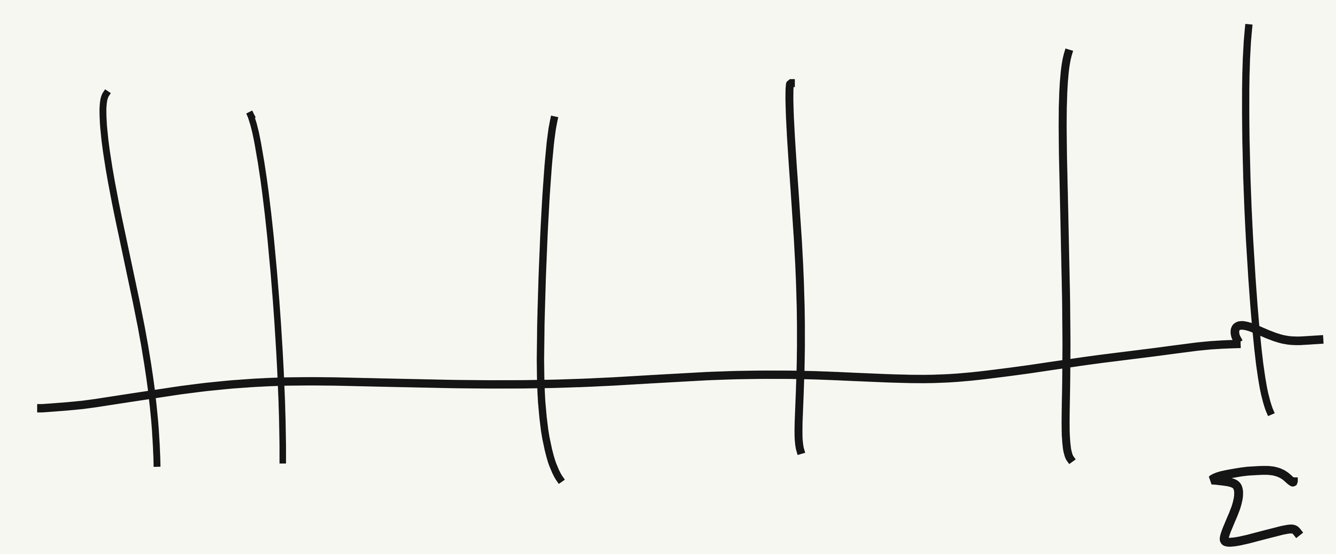
$$\exp\left\{-\frac{1}{k_B}\frac{E}{T}\right\}$$



$\mathbb{R}^2$

$\sigma \in \Sigma$

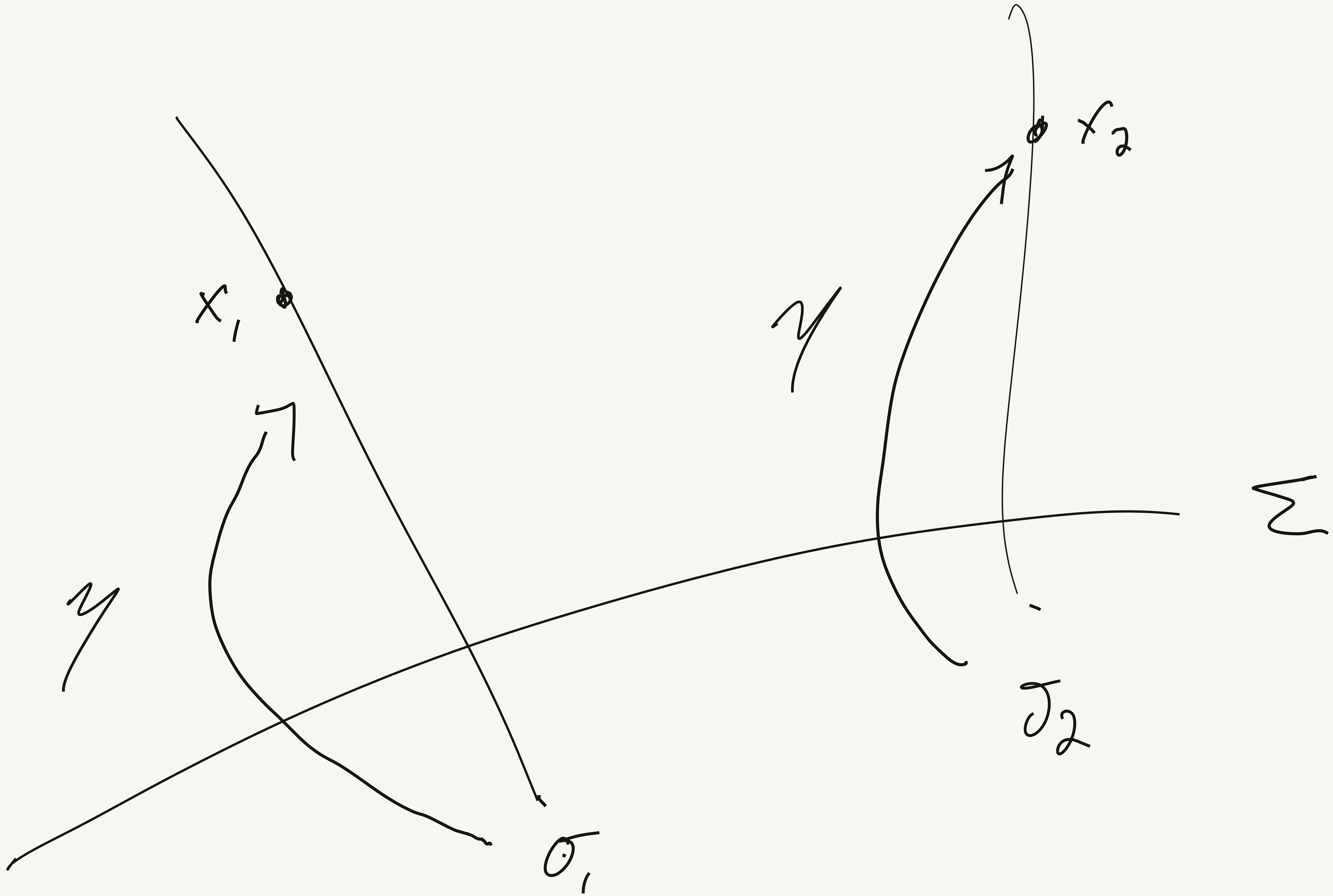
$X \bigsqcup_{\sigma} X_{\sigma_1}$



$x \in X$

$U(1) = S^1$

$e^{i\theta} \in U(1)$



$\mathbb{R}^3 + \mathbb{R}$

$\gamma : \Sigma \to X$

$\psi(x) =$ 

$\gamma : \Sigma \to X$

$\gamma(\sigma) = x$

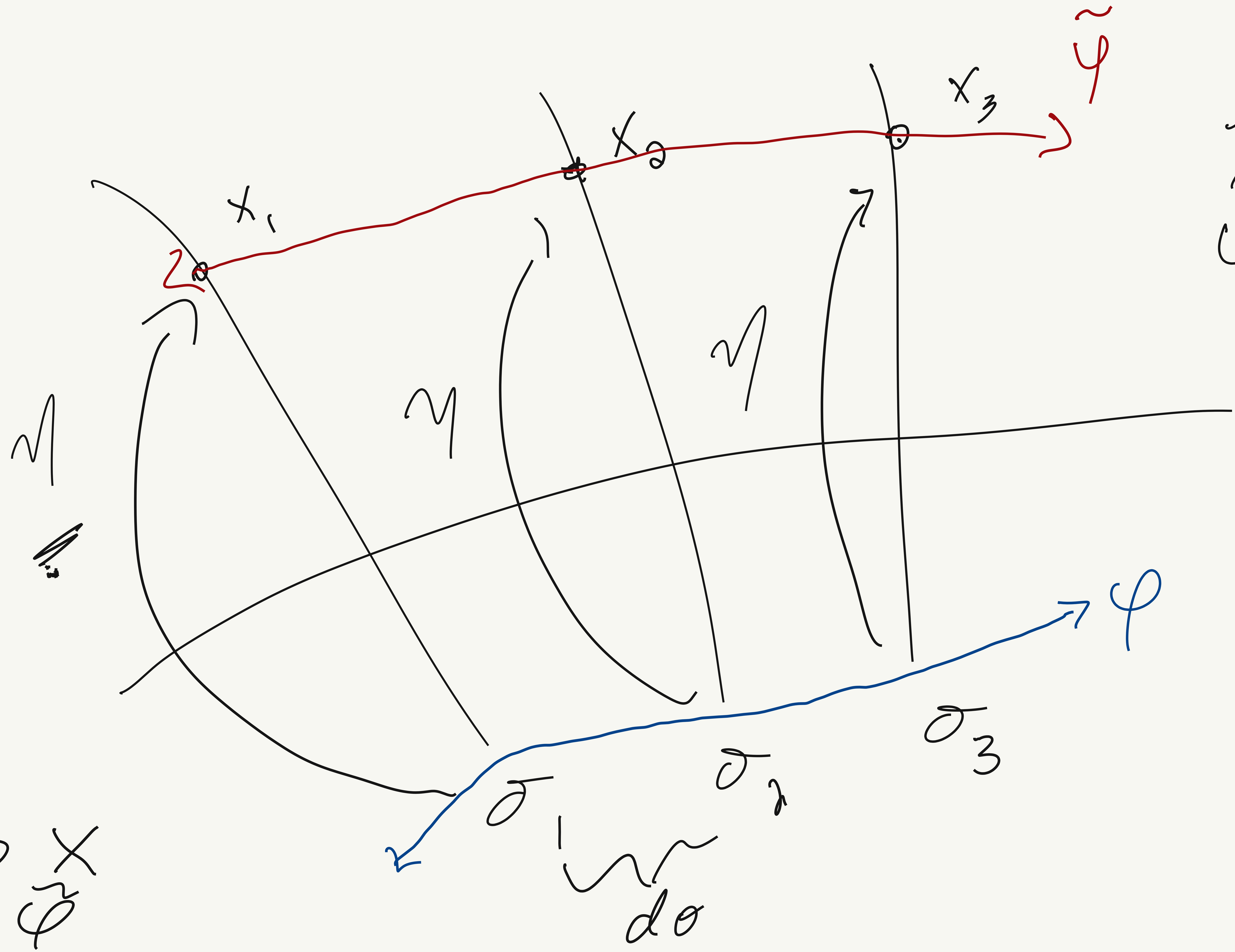
$\gamma \circ \varphi$

$\gamma(\varphi)$

$I \to \Sigma$

$\Sigma \to X$

$\gamma(\varphi) = \tilde{\varphi}$



$\gamma : \Sigma \to X$

$\varphi : I \to \Sigma$

$[0,1]$

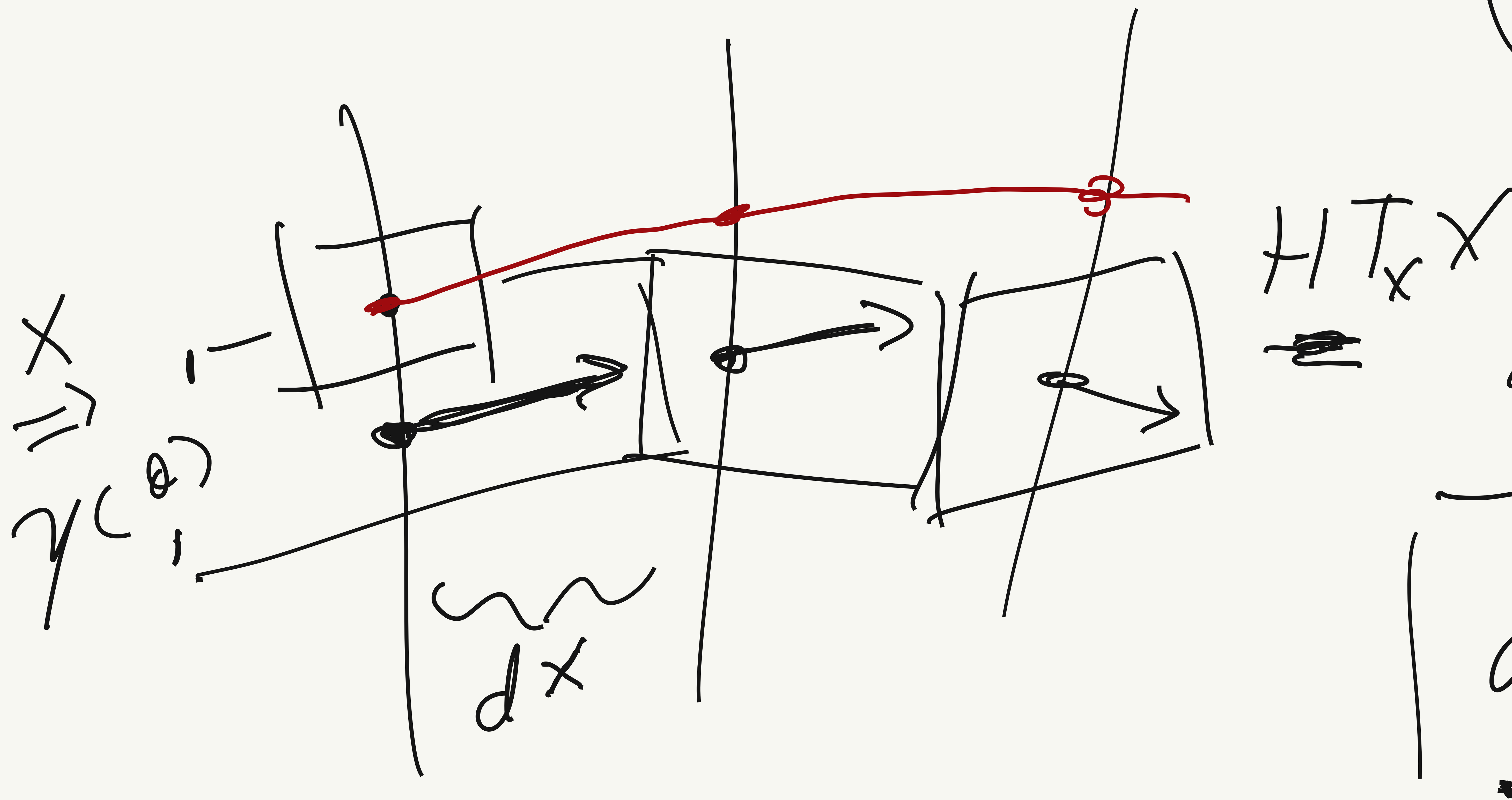
$\mathbb{R}$

$\tilde{\varphi} =$

$I \to X$

(3)

$$\gamma(\theta) = X$$



$$\gamma \circ \varphi = \tilde{\varphi}$$

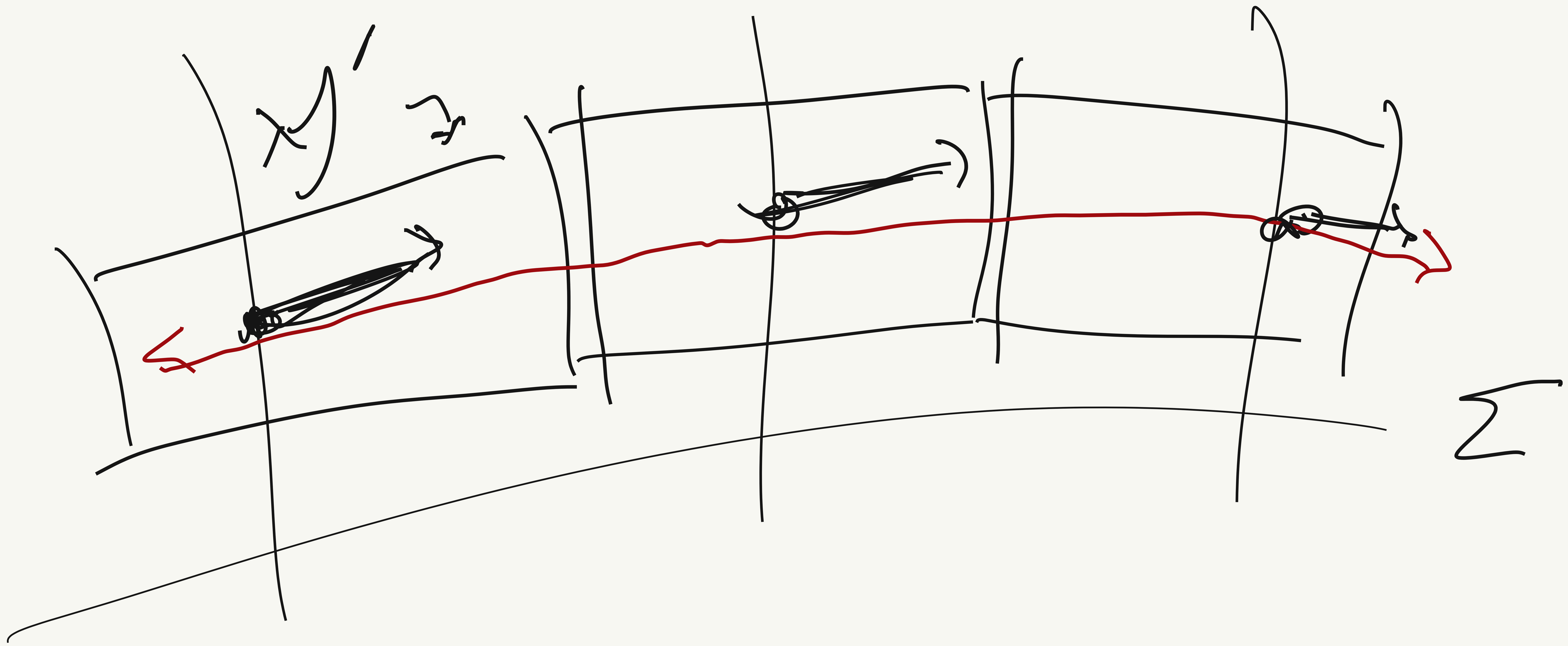
$$\boxed{d\gamma : T\Sigma \longrightarrow T_{\gamma(\theta)}X}$$

$$\tilde{\varphi}'(x) =$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$dx$$

$$v \in T_\sigma \Sigma \quad \forall \sigma$$



$\nabla := HT_x X$

$d\gamma \mapsto T_x X$

$\tilde{\varphi}'$

$=$

$\gamma \circ \varphi$

= Observable

= unknown data generating process

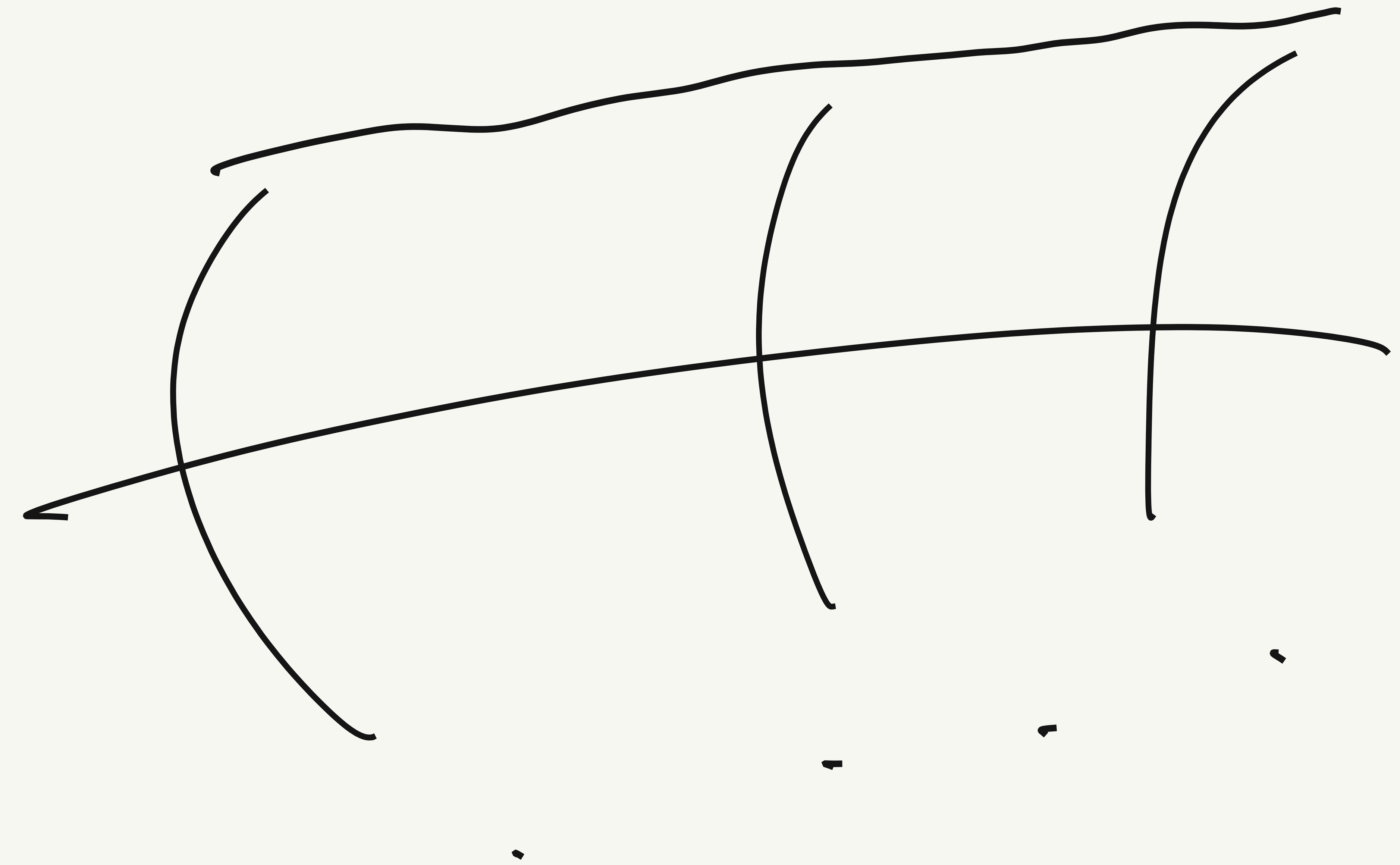
$\gamma \mid \jmath$

$$\nabla := HT_x X$$

$$\longrightarrow \tilde{\varphi}'(x) = -\lambda \jmath'(x) \longleftarrow \text{specifies possible tangent vectors}$$

$$x = \gamma = \{ x(t) \dots x(t_f) \}$$

$-\times\gamma' = \tilde{\varphi}'\nabla \nearrow \tilde{\varphi}$



$-\lambda\gamma' \Rightarrow D_{\tilde{\varphi}'}\gamma = 0$

$U_\alpha \Rightarrow \gamma_\alpha^{*}\nabla \Rightarrow A_\alpha \quad A = \sum_\alpha A_\alpha \, du^\alpha$

$u \in U_\alpha$

$\nabla(\gamma)_{(\gamma : \Sigma \to X)} \mapsto X \xrightarrow{\nabla} A$

$$\underline{\eta'_\alpha} = \underbrace{-A_\alpha(\varphi)}_{\eta^*\nabla}\,\underline{\eta_a}$$

$$\eta = \exp\left\{-\int_\Sigma \underbrace{A_\alpha(\varphi)}_{\eta^*\nabla}\right\}$$

$$\eta = \exp\left\{-\int_\Sigma \lambda\, j'\right\}$$

$$\eta = \exp\{-\lambda \eta\} \qquad (4)$$

$$\mathrm{tra}(\eta)$$

$$\int e^{S_{\mathrm{kin}}} \cdot \mathrm{tra}(\eta)\, d\eta$$

$$\in \Omega(\mathrm{Mom}(\eta))$$

$$d\eta \wedge \mathrm{tra}(\eta)$$

$$J = -\int P(y)\ln(P(y))\,dy$$
$$-\lambda\left(\int P(y)\,dy - 1\right)$$

$$J_h \geq F(y)_h \qquad -\mu\left(\int C(y)\,P(y)\,dy - C\right)$$

$$\hat{}\; E(J(y))$$

$$-\nu\left(\int W(y)\,P(y)\,dy - \varepsilon\right)$$
$$-\rho\left(\int H(y)\,P(y)\,dx + \varepsilon\right)$$

$$0 = \frac{\partial S}{P(\eta)} = \ln(P(\eta)) - \lambda \cdot g$$

$$e^{(-\lambda \cdot g)}$$

$$\langle g \rangle = g$$